ADDITIONAL MATHEMATICS
FORM 5
MODULE 3

LINEAR LAW
# CHAPTER 2: LINEAR LAW

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CHAPTER 2: LINEAR LAW

LINEAR LAW

Linear Relation

Non Linear Relation

Reducing non-linear relation to linear form

Lines of best fit

Determining values of

Formation of equation

Obtaining information
3.1 UNDERSTAND AND USE THE CONCEPT OF LINES OF BEST FIT

THE CONCEPT OF LINES OF BEST FIT.

The properties of the line of best fit.
- The straight line is drawn such away that it passes through as many points as possible.
- The number of points that do not lie on the straight line drawn should be more or less the same both sides of the straight line.

Example.

Draw the line of best fit for the given graph below.

1. Draw the line of best fit for the following diagrams.

Exercises 1
3.1.2 Write equations for lines of best fit

Exercises 2

Example 1

\[ Y = mX + c \]

Gradient, \( m = \frac{8 - 2}{3 - 0} = 2 \)

Y-intercept = 2

Therefore, \( y = 2x + 2 \)

Example 2

\[ Y = mX + c \]

Gradient, \( m = \frac{7 - 3}{3 - 1} = 2 \)

The straight line also passes through the Point (1, 3)

Another equation can be formed is,

\[ 3 = 2(1) + c \]

\[ c = 1 \]

Therefore, \( y = 2x + 1 \)

a) \[ y \]

\[ 9 \]

\[ x (8, 1) \]

b) \[ y \]

\[ x (3, 6) \]
3.1.3 Determine values of variables from:

**a) lines of best fit**

**Example 1**

Gradient = $\frac{4 - 2}{2 - 0} = 1$

Therefore, $\frac{k - 4}{7 - 2} = 1$

$k - 2 = 5$

$k = 7$

**b) Equations of lines of best fit**

**Example 1**

The straight line also passes through the point $(3, \ k)$

Another equation that can be formed is

$k = 3(3) + 4$

$k = 9 + 4 = 13$
## Exercises 3

### a) lines of best fit

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>i)</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
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<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
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<tr>
<td>ii)</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
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<tr>
<td>iii)</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
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</tbody>
</table>

### b) Equations of lines of best fit

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>$y = 2x - 3$</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
</tr>
<tr>
<td>ii)</td>
<td>$y = 6 - x$</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
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<tr>
<td>iii)</td>
<td>$y = x + 2$</td>
<td><img src="http://mathsmozac.blogspot.com" alt="Graph" /></td>
</tr>
</tbody>
</table>

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**Notes:**
- The equations are derived from the points given in the graphs.
- The graphs illustrate the lines of best fit for the given data points.
3.2 APPLY LINEAR LAW TO NON-LINEAR RELATIONS

3.2.1 Reduce non-linear relations to linear form

Example 1

\[ y = ax + \frac{x^2}{b} \]

\[ \frac{y}{x} = a + \frac{x}{xb} \]

\[ \frac{y}{x} = a + \frac{1}{b} \]

\[ Y = c + Xm \]

Example 1

a) \( y = px^q \)

\[ \log_{10} y = \log_{10} (px^q) \]

\[ \log_{10} y = \log_{10} p + q \log_{10} x \]

\[ Y = c + mX \]

Exercises 4

<table>
<thead>
<tr>
<th>Equation</th>
<th>Linear form</th>
<th>Y</th>
<th>X</th>
<th>m</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( y^2 = ax + b )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) ( y = ax^2 + bx )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) ( \frac{a}{y} = \frac{b}{x} + 1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) ( y^2 = 5x^2 + 3x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e) ( y = 3\sqrt{x} + \frac{5}{\sqrt{x}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f) ( y = ab^x )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g) ( y = \frac{4}{a^2} (x + b)^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.2.4 Determine values of constants of non–linear relations given

**Exercises 5**

**Example 1**

The above figure shows part of a straight line graph drawn to represent the equation

\[ xy = ax^2 + b \]

Find the value of \( a \) and \( b \)

Gradient, \( a = \frac{6 - 2}{11 - 1} \)

\[ a = \frac{4}{10} = \frac{2}{5} \]

Therefore, \( xy = \frac{2}{5} x^2 + b \)

Another equation that can be formed is

\[ (1)(2) = \frac{2}{5} (1)^2 + b \]

\[ b = 2 - \frac{2}{5} = \frac{8}{5} \]

Hence \( a = \frac{2}{5} \), \( b = \frac{8}{5} \)

The above figure shows part of a straight line graph drawn to represent the equation

\[ y = ax^2 + bx \]

Find the value of \( a \) and \( b \),

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http://sahatmozac.blogspot.com
b) The above figure shows part of a straight line graph drawn to represent the equation $y = ax^b$.
Find the value of $a$ and $b$.

c) The above figure shows part of a straight line graph drawn to represent the equation $xy = a + bx$.
Find the value of $a$ and $b$. 
3.3 STEPS TO PLOT A STRAIGHT LINE

3.3.1. Using a graph paper.

**Questions**

<table>
<thead>
<tr>
<th>X</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2</td>
<td>9</td>
<td>20</td>
<td>35</td>
<td>54</td>
</tr>
</tbody>
</table>

The above table shows the experimental values of two variables, x and y. It is know that x and y are related by the equation 

\[ y = px^2 + qx \]

a) Draw the line of best fit for \( \frac{y}{x} \) against x

b) From your graph, find,
   i) the initial velocity
   ii) the acceleration
SOLUTION

STEP 1

Reduce the non-linear
To the linear form

\[ y = px^2 + qx \]

\[ \frac{y}{x} = \frac{px^2}{x} + \frac{qx}{x} \]

\[ \frac{y}{x} = px + q \]

Linear form
\[ Y = mX + c \]

STEP 2

Construct table

<table>
<thead>
<tr>
<th>.x</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>.y</td>
<td>2</td>
<td>9</td>
<td>20</td>
<td>35</td>
<td>54</td>
</tr>
<tr>
<td>( \frac{y}{x} )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

STEP 3

Using graph paper,
- Choose a suitable scale so that the graph drawn is as big as possible.
- Label both axis
- Plot the graph of Y against X and draw the line of best fit

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STEP 4

From the graph, find $m$ and $c$.

Gradient, $m = \frac{9 - 1}{6 - 2} = 2$

Construct a right-angled triangle, so that two vertices are on the line of best fit, calculate the gradient, $m$.

Y-intercept = $c = -3$

Determine the Y-intercept, $c$ from the straight line graph.
Exercises 6

1. The table below shows some experimental data of two related variable x and y. It is known that x and y are related by an equation in the form $y = ax + bx^2$, where a and b are constants.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>7</td>
<td>16</td>
<td>24</td>
<td>24</td>
<td>16</td>
<td>0</td>
<td>-24</td>
</tr>
</tbody>
</table>

a) Draw the straight line graph of $\frac{y}{x}$ against x.

b) Hence, use the graph to find the values of a and b.
2. The table below shows some experimental data of two related variable x and y

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1.67</td>
<td>1.9</td>
<td>2.21</td>
<td>2.41</td>
<td>2.65</td>
<td>2.79</td>
</tr>
</tbody>
</table>

It is known that x and y are related by an equation in the form

\[ y = \frac{ax}{y} + \frac{b}{y}, \]  where a and b are constants.

a) Draw the straight line graph \( y^2 \) against x
b) Hence, use the graph to find the values of a and b
4.0 SPM QUESTIONS

1. SPM 2003 (paper 1, question no 10)

x and y are related by equation \( y = px^2 + qx \), where \( p \) and \( q \) are constants. A straight line is obtained by plotting \( \frac{y}{x} \) against \( x \), as shown in the diagram below.

Calculate the values of \( p \) and \( q \).
2. SPM 2004 (paper 1, question no 13)

Diagram below shows a straight line graph of \( \frac{y}{x} \) against \( x \)

Gigen that \( y = 6x - x^2 \), calculate the value of \( k \) and of \( h \)
3. SPM 2005 (paper 1, question no 13)

The variable $x$ and $y$ are related by the equation $y = kx^4$ where $k$ is a constant.

a) Convert the equation $y = kx^4$ to linear form

b) Diagram below shows the straight line obtained by plotting $\log_{10} y$ against $\log_{10} x$

Find the value of
i) $\log_{10} k$
ii) $h$. 

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4. SPM 2003 (paper 2, question no 7)

Table 1 shows the values of two variables, \( x \) and \( y \), obtained from an experiment. It is known that \( x \) and \( y \) are related by the equation \( y = pk^x \), where \( p \) and \( k \) are constants.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1.59</td>
<td>1.86</td>
<td>2.40</td>
<td>3.17</td>
<td>4.36</td>
<td>6.76</td>
</tr>
</tbody>
</table>

(a) Plot \( \log y \) against \( x^2 \).
Hence, draw the line of best fit
(b) Use the graph in (a) to find value of
(i) \( p \)
(ii) \( k \)
5. SPM 2004 (paper 2, question no 7)

Table 1 shows the values of two variables, \(x\) and \(y\), obtained from an experiment. Variables \(x\) and \(y\) are related by the equation \(y = pk^x\), where \(p\) and \(k\) are constants.

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>3.16</td>
<td>5.50</td>
<td>9.12</td>
<td>16.22</td>
<td>28.84</td>
<td>46.77</td>
</tr>
</tbody>
</table>

(a) Plot \(\log_{10} y\) against \(x\) by using scala of 2 cm to 2 units on the \(x\)-axis and 2 cm to 0.2 unit on the \(\log_{10}\)-axis.
Hence, draw the line of best fit.

(b) Use the graph in (a) to find value of
\[(i) \quad p\]
\[(ii) \quad k\]
6. SPM 2005( paper 2, question no 7)

Table 1 shows the values of two variables, x and y, obtained from an experiment.

The variables x and y are related by the equation \( y = px + \frac{r}{px} \),

where p and r are constants.

<table>
<thead>
<tr>
<th>x</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5.5</td>
<td>4.7</td>
<td>5.0</td>
<td>6.5</td>
<td>7.7</td>
<td>8.4</td>
</tr>
</tbody>
</table>

(b) Plot \( xy \) against \( x^2 \) by using a scale of 2 cm to 5 units on both axes.
Hence, draw the line of best fit

(b) Use the graph in (a) to find the value of

(i) p
(ii) r
The above figure shows part of a straight-line graph drawn to represent the equation 

\[ y = \frac{6}{x} - \frac{1}{2} \]

Find the value of \( h \) and of \( k \)
3. The table below shows some experimental data of two related variable x and y

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>-1.5</td>
<td>9</td>
<td>43.5</td>
<td>114</td>
<td>232.5</td>
</tr>
</tbody>
</table>

It is known that x and y are related by an equation in the form
\[ y = ax^3 - bx \], where a and b are constants.

a) Change the equation to the linear form and hence draw the straight line graph for values of x and y

b) From your graph,
   I). determine the values of a and b
   II). find the value of y when x = 3.5
6.0 ANSWERS

Exercises 2

a) \( y = -x + 9 \)
b) \( y = 3x - 3 \)
c) \( y = -x + 7 \)
d) \( y = 2x - 8 \)

Exercises 3

a) i) \( k = 5 \)  ii) \( k = 3 \)
     iii) \( k = 5 \)

Exercises 4

\[
\begin{array}{cccc}
Y & X & m & c \\
\hline
\text{a)} & y^2 & x & a & b \\
\text{b)} & y & x & a & b \\
\text{c)} & \frac{1}{y} & \frac{1}{x} & \frac{b}{a} & \frac{1}{a} \\
\text{d)} & \frac{y^2}{x} & x & 5 & 3 \\
\text{e)} & y \sqrt{x} & x & 3 & 5 \\
\text{f)} & \log y & x & \log b & \log a \\
\text{g)} & \sqrt{y} & x & \frac{2}{a} & \frac{2b}{a} \\
\end{array}
\]

Exercises 5

a) \( a = 1, \ b = 2 \)
b) \( a = 11, \ b = 8 \)
c) \( a = 3, \ b = 3 \)

Exercises 5

1) \( a = -1, \ b = 10 \)
2) \( a = 0.5, \ b = 2.8 \)

SPM Questions

1) \( p = -2, \ q = 13 \)
2) \( h = 3, \ k = 4 \)
3) a) \( \log y = 4 \log x + \log k \)
     b) \( k = 100, \ h = 11 \)
4) \( p = 1.259, \ k = 1.109 \)
5) \( p = 1.82, \ k = 1.307 \)
6) \( P = 1.37, \ r = 5.48 \)

Assessment test

1) \( h = 6, \ k = 2 \)
2) a) \( \frac{y}{x} = ax^2 - b \)
     b) i. \( a = 2, \ b = 3.6 \)
     ii. \( y = 73.5 \)